Hydrostatic Forces on Surfaces

This chapter deals with the fluids (*i.e.*, liquids and gases) at rest. This means that there will be no relative motion between adjacent or neighbouring fluid layers. The velocity gradient, which is equal to the change of velocity between two adjacent fluid layers divided by the distance between the layers, will be zero or $\frac{du}{dy} = 0$. The shear stress which is equal to $\mu \frac{\partial u}{\partial y}$ will also be zero. Then the forces acting on the fluid particles will be :

1. due to pressure of fluid normal to the surface,
2. due to gravity (or self-weight of fluid particles).

TOTAL PRESSURE AND CENTRE OF PRESSURE

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.1.

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

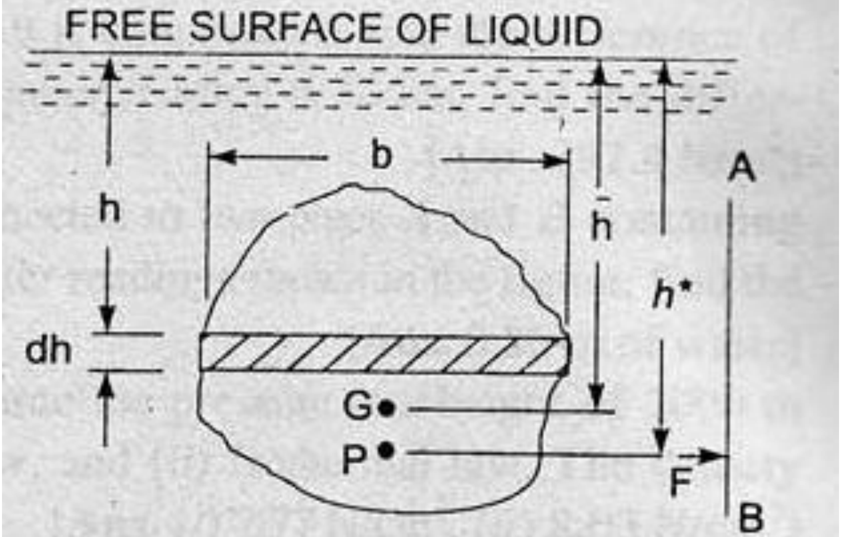


Fig. 3.1

(*a*) **Total Pressure (F).** The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

Pressure intensity on the strip, $p = \rho g h$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho g h \times b \times dh$

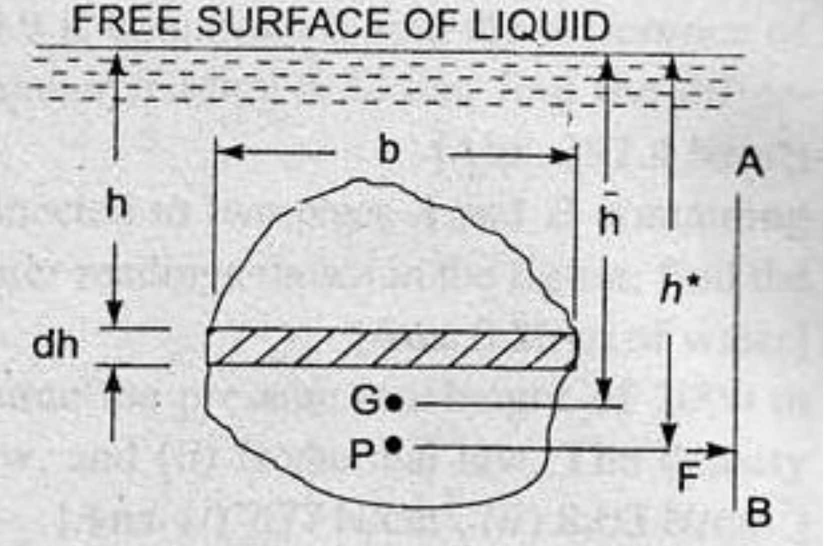


Fig. 3.1

Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

$$\int b \times h \times dh = \int h \times dA$$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$$\therefore \quad F = \rho g A \bar{h}$$

For water the value of $\rho = 1000\ kg/m^3$ and $g = 9.81\ m/s^2$. The force will be in Newton.

(*b*) **Centre of Pressure (h^*).** Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$$= dF \times h \qquad \{\because \; dF = \rho g h \times b \times dh\}$$

$$= \rho g h \times b \times dh \times h$$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \; bdh = dA)$$

$$\int h^2 dA = \int b h^2 dh$$

= Moment of Inertia of the surface about free surface of liquid

$$= I_0$$

Sum of moments about free surface

$$= \rho g I_0$$

$$F \times h^* = \rho g I_0$$

$$F = \rho g A \bar{h}$$

$$\rho g A \bar{h} \times h^* = \rho g I_0$$

$$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}}$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \overline{h^2}$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_G in equation

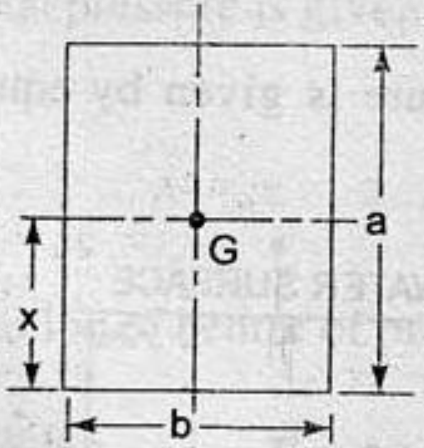
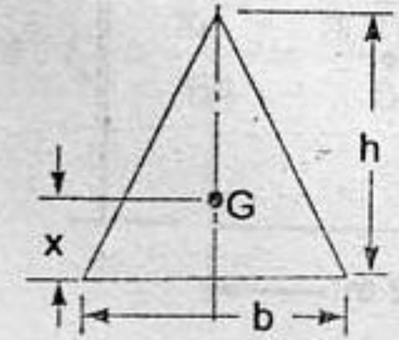
$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

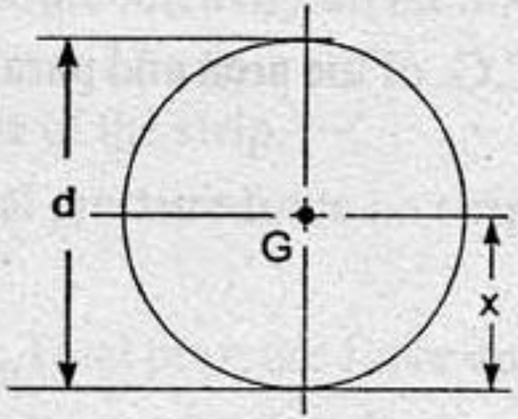
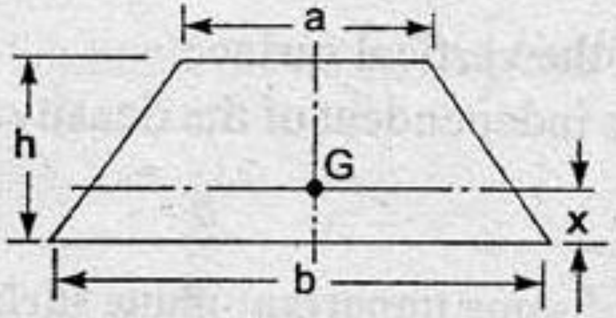
$\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid.

Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

Table 3.1 The moments of inertia and other geometric properties of some important plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Problem 3.1 *A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

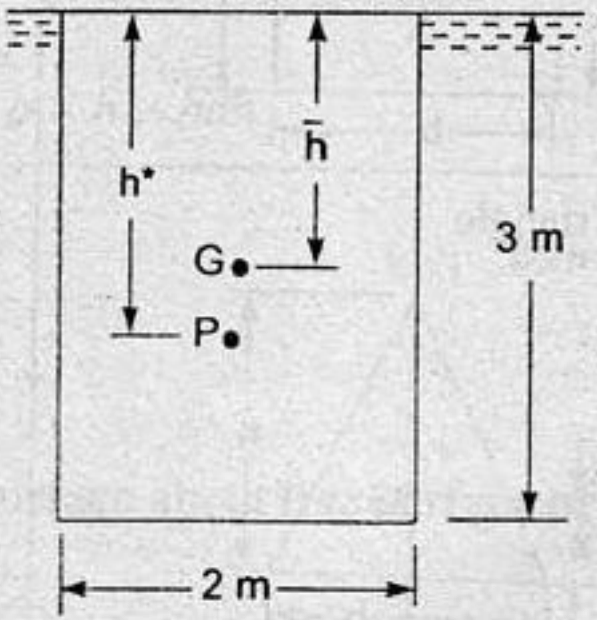
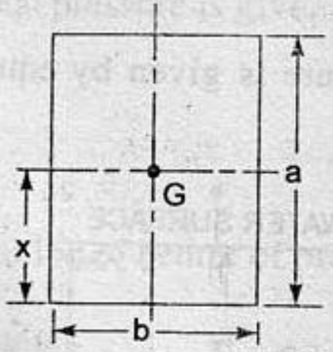


Fig. 3.2

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$

(*a*) **Upper edge coincides with water surface (Fig. 3.2).** Total pressure is given by equation (3.1) as

$$F = \rho g A \bar{h}$$

$\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$

$A = 3 \times 2 = 6 \text{ m}^2$, $\bar{h} = \frac{1}{2}(3) = 1.5 \text{ m}.$

$$F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= \mathbf{88290 \text{ N}.}$$

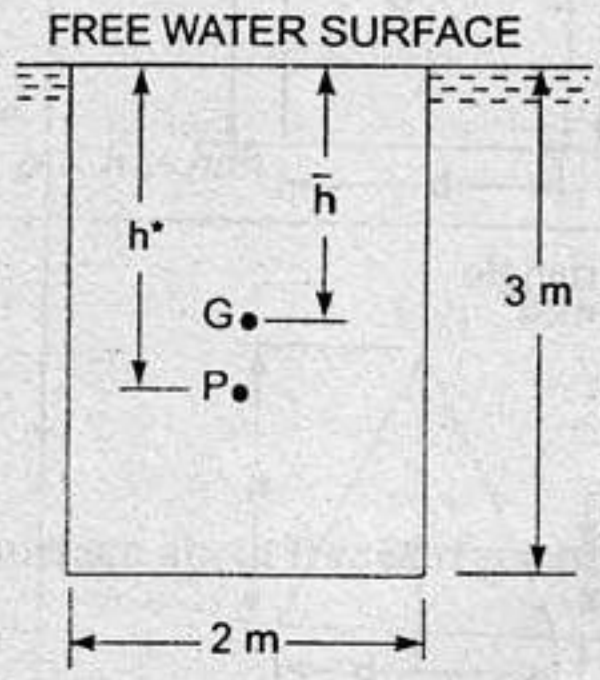


Fig. 3.2

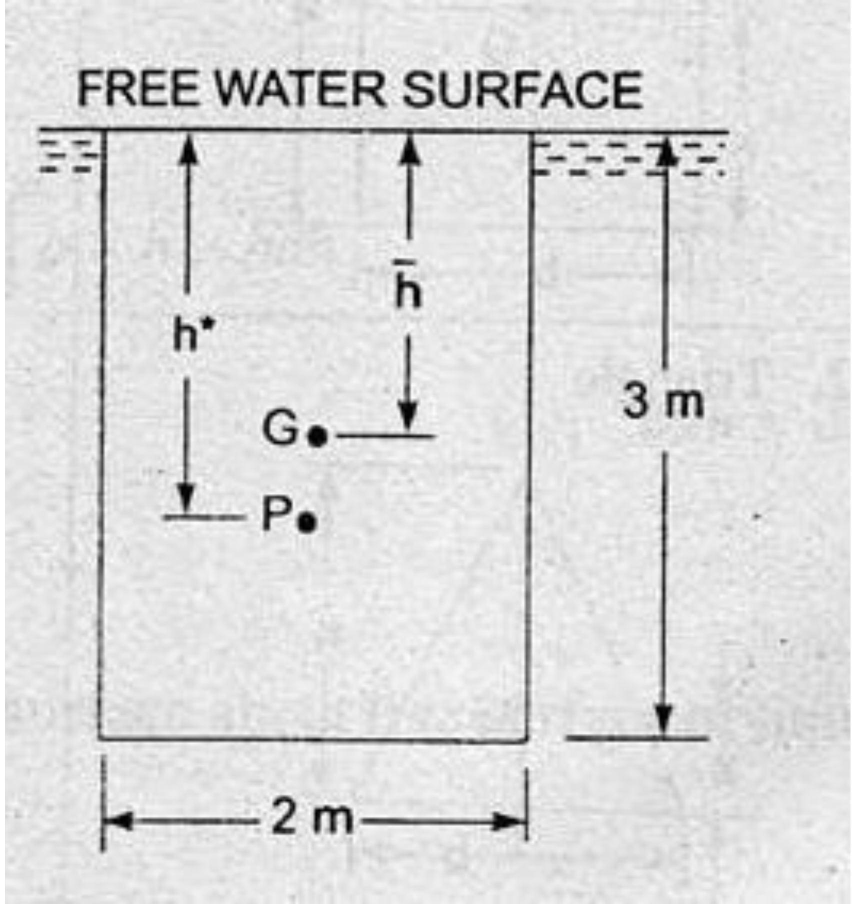
Depth of centre of pressure is given by equation

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0\ m.}$$



(*b*) **Upper edge is 2.5 m below water surface (Fig. 3.3).** Total pressure (F) is given by

$$F = \rho g A \bar{h}$$

$\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$$F = 1000 \times 9.81 \times 6 \times 4.0 = \mathbf{235440\ N.}$$

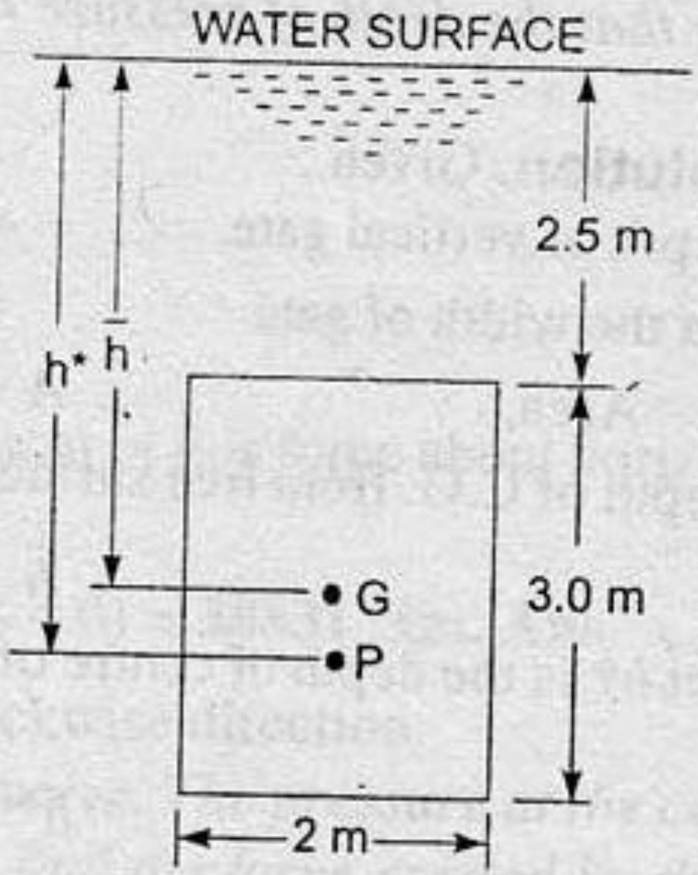


Fig. 3.3

Centre of pressure is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$$\therefore \quad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$= 0.1875 + 4.0 = 4.1875 =$ **4.1875 m.**

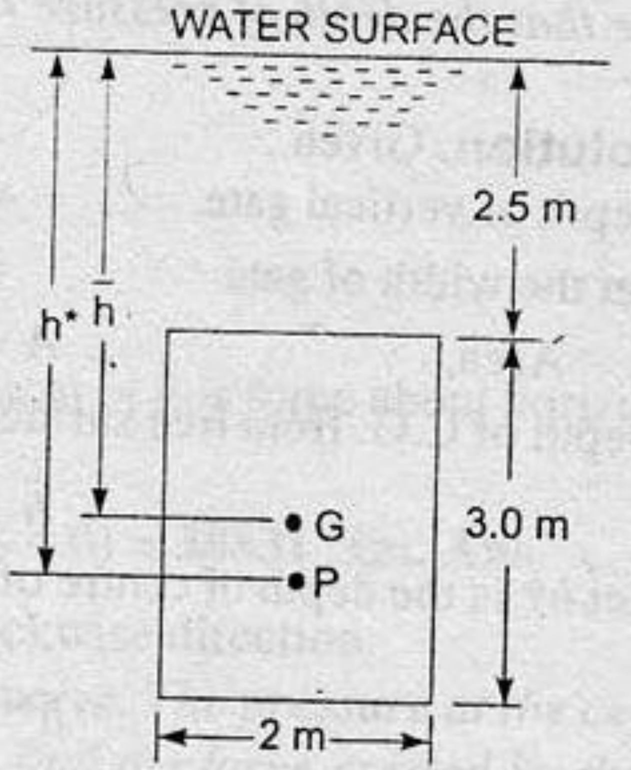


Fig. 3.3

Problem 3.2 *Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.*

Given : Dia. of plate, $d = 1.5$ m

$$A = \frac{\pi}{4}(1.5)^2 = 1.767 \text{ m}^2$$

$$\bar{h} = 3.0 \text{ m}$$

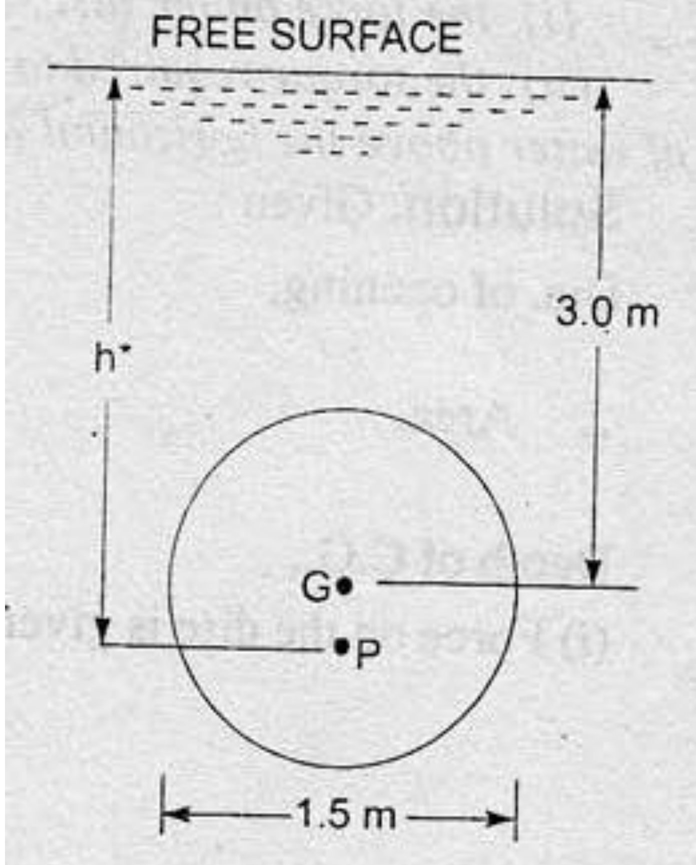


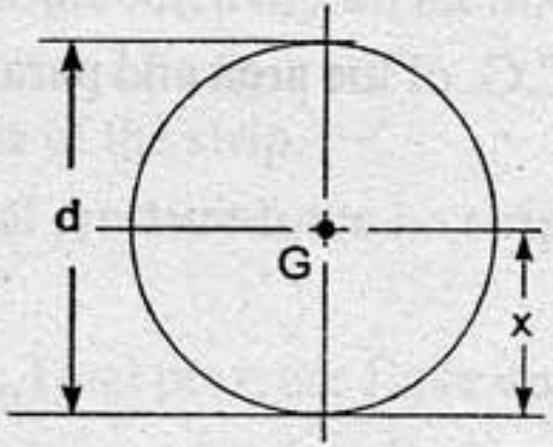
Fig. 3.4

Total pressure is given by equation $F = \rho g A \bar{h}$

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$

$$= \mathbf{52002.81\ N}$$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—

Position of centre of pressure (h^*) is given by equation (3.5)

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi d^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.2485 \text{ m}^4$

$$h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$

$= \mathbf{3.0468\ m.}$

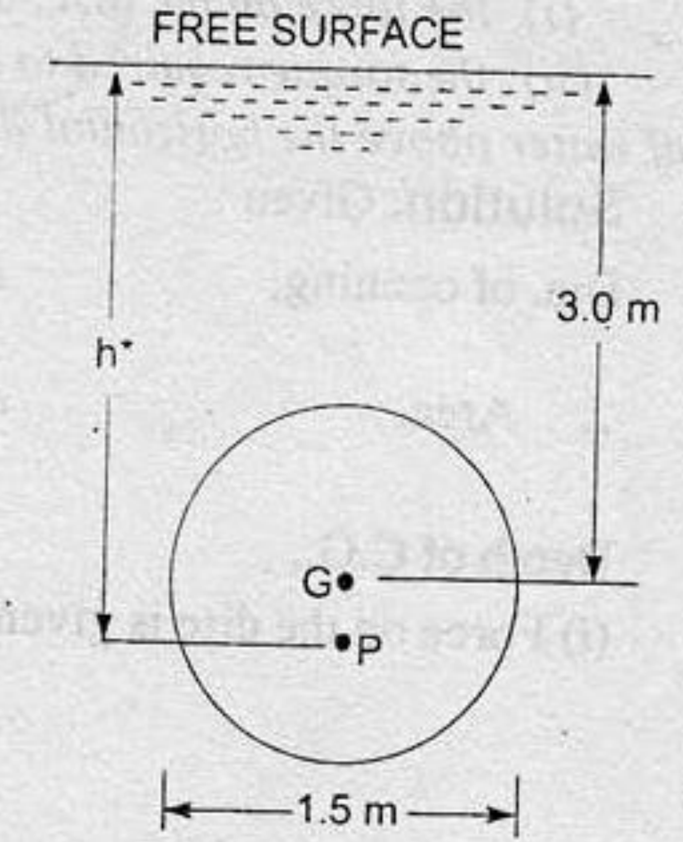


Fig. 3.4

Problem 3.3 *A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 'd' metres in length and depth of centroid of the area is 'p' m below the water surface.*

Prove that the depth of pressure is equal to $\left(p + \dfrac{d^2}{12p}\right)$.

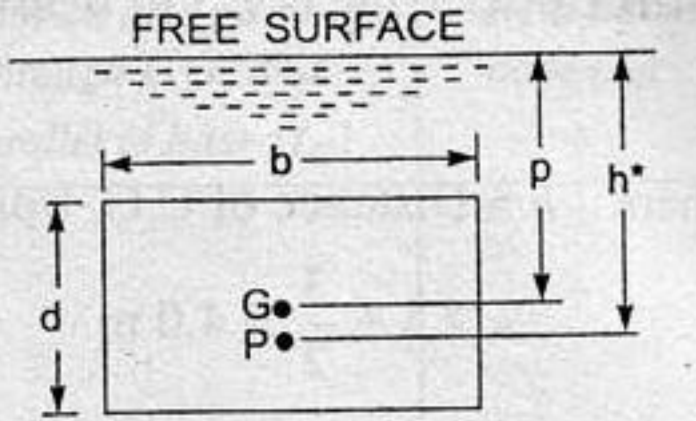
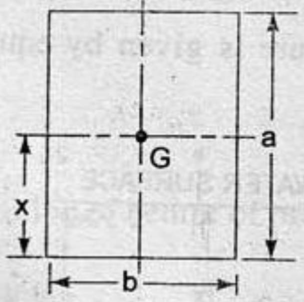


Fig. 3.5

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
1. Rectangle 	$x = \dfrac{d}{2}$	bd	$\dfrac{bd^3}{12}$	$\dfrac{bd^3}{3}$

Depth of vertical gate $= d$ m

Let the width of gate $= b$ m

$\therefore$ Area, $A = b \times d$ m^2

Depth of C.G. from free surface

$$\bar{h} = p \text{ m.}$$

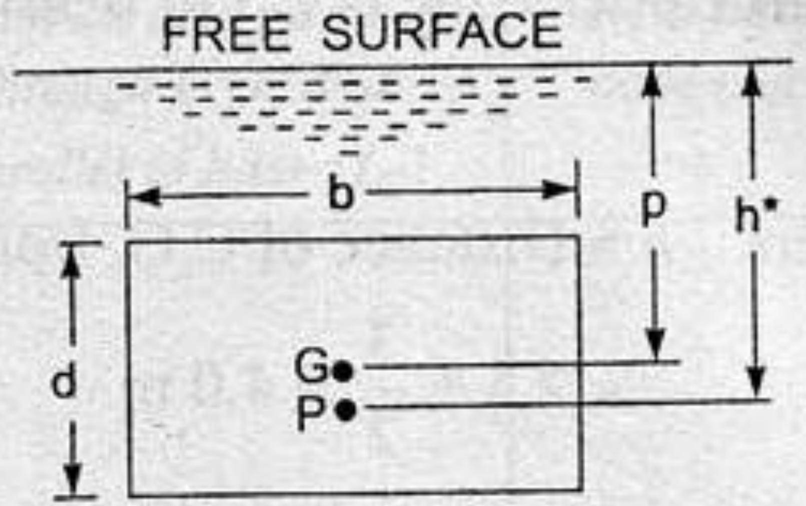


Fig. 3.5

Let h^* is the depth of centre of pressure from free surface, which is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + h, \text{ where } I_G = \frac{bd^2}{12}$$

$$h^* = \left(\frac{bd^3}{12} \bigg/ b \times d \times p \right) + p = \frac{d^2}{12\,p} + p \quad \text{or} \quad \mathbf{p + \frac{d^2}{12}}.$$

Problem 3.4 *A circular opening, 3 m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter. Calculate :*

the force on the disc,

Dia. of opening; $d = 3$ m

$\therefore$ Area, $A = \frac{\pi}{4} \times 3^2 = 7.0685 \text{ m}^2.$

Depth of C.G., $\bar{h} = 4$ m

(*i*) Force on the disc is given by equation (3.1) as

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 7.0685 \times 4.0$$

$$= \mathbf{277368 \ N = 277.368 \ kN}$$

Problem 3.5 *A pipe line which is 4 m in diameter contains a gate valve. The pressure at the centre of the pipe is 19.6 N/cm². If the pipe is filled with oil of sp. gr. 0.87, find the force exerted by the oil upon the gate and position of centre of pressure.* (Converted to SI Units, A.M.I.E., Winter, 1975)

Dia. of pipe, $d = 4$ m

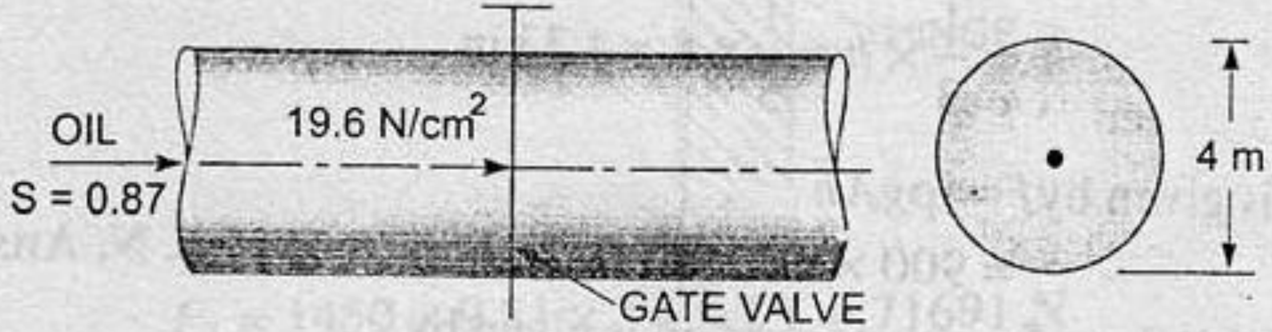


Fig. 3.7

Area, $A = \frac{\pi}{4} \times 4^2 = 4\pi \text{ m}^2$

Sp. gr. of oil, $S = 0.87$

∴ Density of oil, $\rho_0 = 0.87 \times 1000 = 870 \text{ kg/m}^3$

∴ Weight density of oil, $w_0 = \rho_0 \times g = 870 \times 9.81 \text{ N/m}^3$

Pressure at the centre of pipe, $p = 19.6 \text{ N/cm}^2 = 19.6 \times 10^4 \text{ N/m}^2$

∴ Pressure head at the centre $= \dfrac{p}{w_0} = \dfrac{19.6 \times 10^4}{870 \times 9.81} = 22.988 \text{ m}$

∴ The height of equivalent free oil surface from the centre of pipe = 22.988 m.

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988$ m.

Now the force exerted by the oil on the gate is given by

$$F = \rho g A \bar{h}$$

ρ = density of oil = 870 kg/m^3

$$F = 870 \times 9.81 \times 4\pi \times 22.988 = \mathbf{2465500\ N = 2.465\ MN}$$

Position of centre of pressure (h^*) is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times \bar{h}} + \bar{h} = \frac{d^2}{16\bar{h}} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988$$

$$= 0.043 + 22.988 = \mathbf{23.031\ m}$$

centre of pressure is below the centre of the pipe by a distance of 0.043 m.

Problem 3.6 *Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4 m and altitude 4 m when it is immersed vertically in an oil of sp. gr. 0.9. The base of the plate coincides with the free surface of oil.*

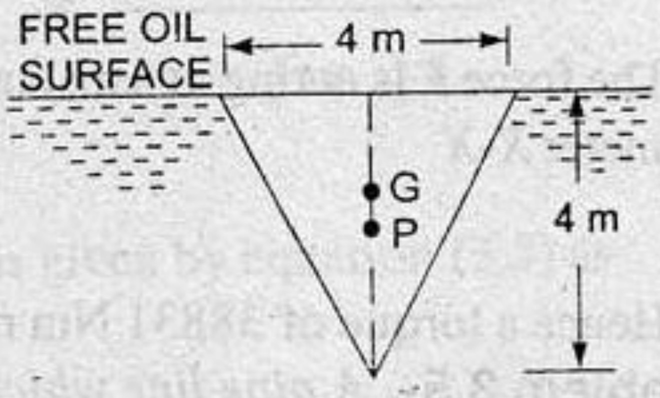


Fig. 3.8

Solution. Given :

Base of plate, $b = 4$ m

Height of plate, $h = 4$ m

$\therefore$ Area, $A = \dfrac{b \times h}{2} = \dfrac{4 \times 4}{2} = 8.0 \text{ m}^2$

Sp. gr. of oil, $S = 0.9$

$\therefore$ Density of oil, $\rho = 900 \text{ kg/m}^3$.

The distance of C.G. from free surface of oil,

$$\bar{h} = \frac{1}{3} \times h = \frac{1}{3} \times 4 = 1.33 \text{ m.}$$

Total pressure (F) is given by $F = \rho g A \bar{h}$

$$= 900 \times 9.81 \times 8.0 \times 1.33 \text{ N} = \mathbf{9597.6.\ N.}$$

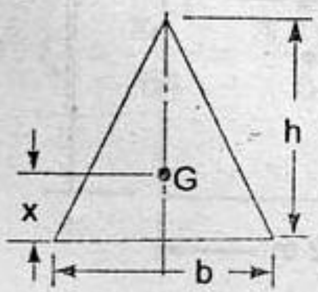
Centre of pressure (h^*) from free surface of oil is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of triangular section about its C.G.

$$= \frac{bh^3}{36} = \frac{4 \times 4^3}{36} = 7.11 \text{ m}^4$$

$$\therefore \quad h^* = \frac{7.11}{8.0 \times 1.33} + 1.33 = 0.6667 + 1.33 = \mathbf{1.99\ m}$$

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Problem 3.7 *A vertical sluice gate is used tc cover an opening in a dam. The opening is 2 m wide and 1.2 m high. On the upstream of the gate, the liquid of sp. gr. 1.45, lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.* (A.M.I.E., May, 1975)

Width of gate, $b = 2$ m

Depth of gate, $d = 1.2$ m

∴ Area, $A = b \times d = 2 \times 1.2 = 2.4$ m^2

Sp. gr. of liquid $= 1.45$

∴ Density of liquid, $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^3

Let F_1 = Force exerted by the fluid of sp. gr. 1.45 on gate

F_2 = Force exerted by water on the gate.

The force F_1 is given by $F_1 = \rho_1 g \times A \times \bar{h}_1$

where $\rho_1 = 1.45 \times 1000 = 1450$ kg/m^2

$\bar{h}_1$ = Depth of C.G. of gate from free surface of liquid

$$= 1.5 + \frac{1.2}{2} = 2.1 \text{ m.}$$

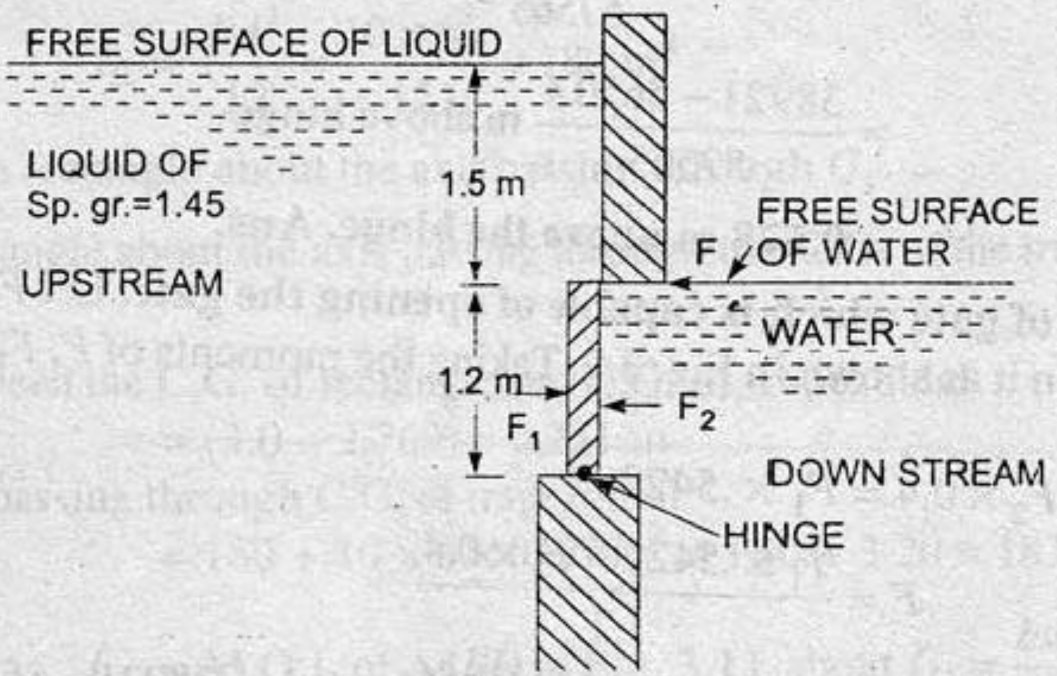


Fig. 3.9

$$F_1 = 1450 \times 9.81 \times 2.4 \times 2.1 = 71691 \text{ N}$$

Similarly, $F_2 = \rho_2 g.A\bar{h}_2$

where $\rho_2 = 1{,}000$ kg/m^3

$\bar{h}_2$ = Depth of C.G. of gate from free surface of water

$= \frac{1}{2} \times 1.2 = 0.6$ m

$\therefore \quad F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126$ N

Resultant force on the gate $= F_1 - F_2 = 71691 - 14126 =$ **57565 N.**

(*ii*) **Position of centre of pressure of resultant force.** The force F_1 will be acting at a depth of h_1^* from free surface of liquid, given by the relation

$$h_1^* = \frac{I_G}{A\bar{h}_1} + \bar{h}_1$$

$$I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 \text{ m}^4$$

$$h_1^* = \frac{.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571 \text{ m}$$

Distance of F_1 from hinge

$$= (1.5 + 1.2) - h_1^* = 2.7 - 2.1571 = 0.5429 \text{ m}$$

The force F_2 will be acting at a depth of h_2^* from free surface of water

$$h_2^* = \frac{I_G}{A\bar{h}_2} + \bar{h}_2$$

$I_G = 0.288 \text{ m}^4$, $\bar{h}_2 = 0.6$ m, $A = 2.4 \text{ m}^2$.

$$h_2^* = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8 \text{ m}$$

Distance of F_2 from hinge $= 1.2 - 0.8 = 0.4$ m

The resultant force 57565 N will be acting at a distance given by

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{57565}$$

$$= \frac{38921 - 5650.4}{57565} \text{ m above hinge}$$

$$= \mathbf{0.578 \text{ m above the hinge}}$$

(*iii*) **Force at the top of gate which is capable of opening the gate.** Let F is the force required on the top of the gate to open it as shown in Fig. 3.9. Taking the moments of F, F_1 and F_2 about the hinge, we get

$$F \times 1.2 + F_2 \times 0.4 = F_1 \times .5429$$

or

$$F = \frac{F_1 \times .5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{1.2} = \frac{38921 - 5650.4}{1.2}$$

$$= \mathbf{27725.5\ N}$$

Problem 3.8 *A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 10 m wide at the bottom and 6 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is just level with the top and dock is empty.*

Solution. Given :

Width at top = 16 m

Width at bottom = 10 m

Depth, $d = 6$ m

Area of trapezoidal *ABCD*,

$$A = \frac{(BC + AD)}{2} \times d$$

$$= \frac{(10+16)}{2} \times 6 = 78 \text{ m}^2$$

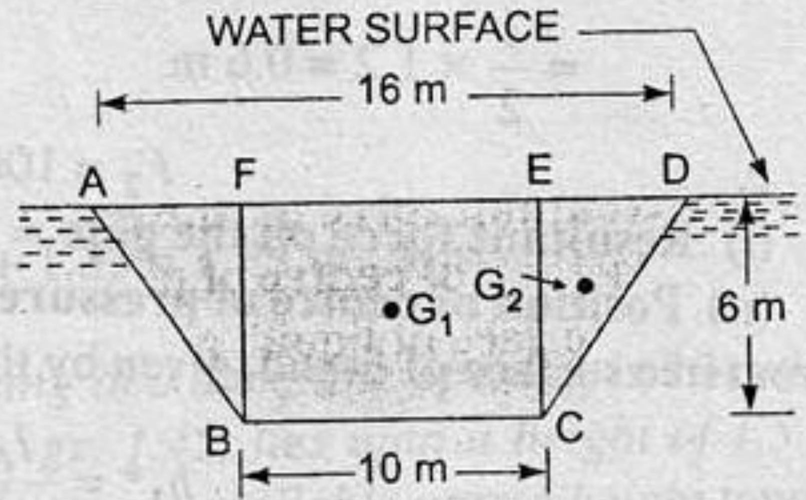


Fig. 3.10

Depth of C.G. of trapezoidal area *ABCD* from free surface of water,

$$\bar{h} = \frac{10 \times 6 \times 3 + \frac{(16-10)}{2} \times 6 \times \frac{1}{3} \times 6}{78}$$

$$= \frac{180 + 36}{78} = 2.769 \text{ m from water surface}$$

(*i*) **Total Pressure (F).** Total pressure, F is given by

$$F = \rho g A \bar{h} = 1000 \times 9.81 \times 78 \times 2.769 \text{ N} \quad = 2118783 \text{ N} = \mathbf{2.118783 \text{ MN}}.$$

(*ii*) **Centre of Pressure (h*).** Centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of trapezoidal *ABCD* about its C.G.

Let I_{G_1} = M.O.I. of rectangle *FBCE* about its C.G.

I_{G_2} = M.O.I. of two Δs *ABF* and *ECD* about its C.G.

Then $$I_{G_1} = \frac{bd^3}{12} = \frac{10 \times 6^3}{12} = 180 \text{ m}^4$$

I_{G_1} is the M.O.I. of the rectangle about the axis passing through G_1.

∴ M.O.I. of the rectangle about the axis passing through the C.G. of the trapezoidal I_{G_1} + Area of rectangle $\times x_1^2$

where x_1 is distance between the C.G. of rectangle and C.G. of trapezoidal

$$= (3.0 - 2.769) = 0.231 \text{ m}$$

∴ M.O.I. of *FBCE* passing through C.G. of trapezoidal

$$= 180 + 10 \times 6 \times (0.231)^2 = 180 + 3.20 = 183.20 \text{ m}^4$$

$$I_{G_2} = \text{M.O.I. of } \Delta ABD \text{ in Fig. 3.11 about } G_2 = \frac{bd^3}{36}$$

$$= \frac{(16-10)\times 6^3}{36} = 36 \text{ m}^4$$

The distance between the C.G. of triangle and C.G. of trapezoidal

$$= (2.769 - 2.0) = 0.769$$

$\therefore$ M.O.I. of the two Δs about an axis passing through C.G. of trapezoidal

$$= I_{G_2} + \text{Area of triangles} \times (.769)^2$$

$$= 36.0 + \frac{6 \times 6}{2} \times (.769)^2$$

$$= 36.0 + 10.64 = 46.64$$

$\therefore$ I_G = M.O.I. of trapezoidal about its C.G.

= M.O.I. of rectangle about the C.G. of trapezoidal

+ M.O.I. of triangles about the C.G. of the trapezoidal

$= 183.20 + 46.64 = 229.84 \text{ m}^4$

$$\therefore \quad h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

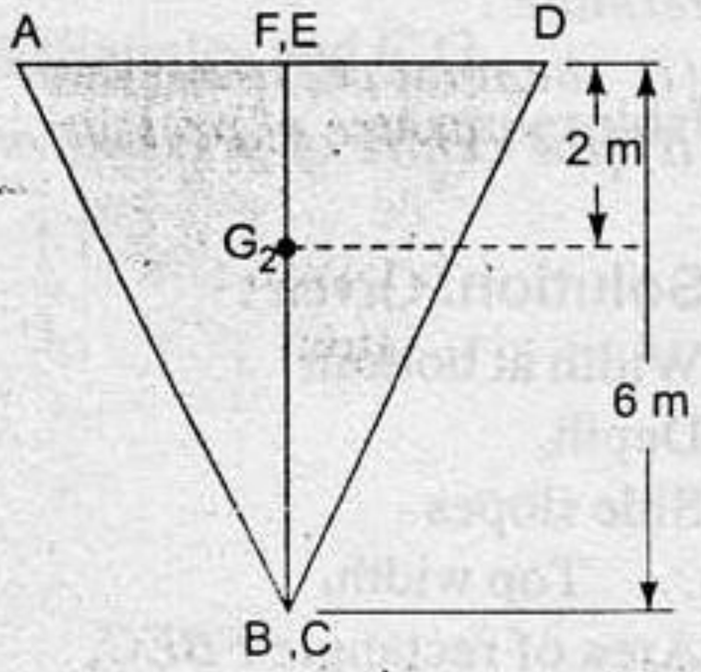


Fig. 3.11

$A = 78,\ \bar{h} = 2.769$

$$h^* = \frac{229.84}{78 \times 2.769} + 2.769 = 1.064 + 2.769 = \mathbf{3.833\ m.\ Ans.}$$

Centre of Pressure

$$(h^*) = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$I_G = \frac{\left(a^2 + 4ab + b^2\right)}{36(a+b)} \times h^3 = \frac{\left(10^2 + 4 \times 10 \times 16 + 16^2\right)}{36(10+16)} \times 6^3$$

$$= \frac{(100 + 640 + 256)}{36 \times 26} \times 216 = 229.846 \text{ m}^4$$

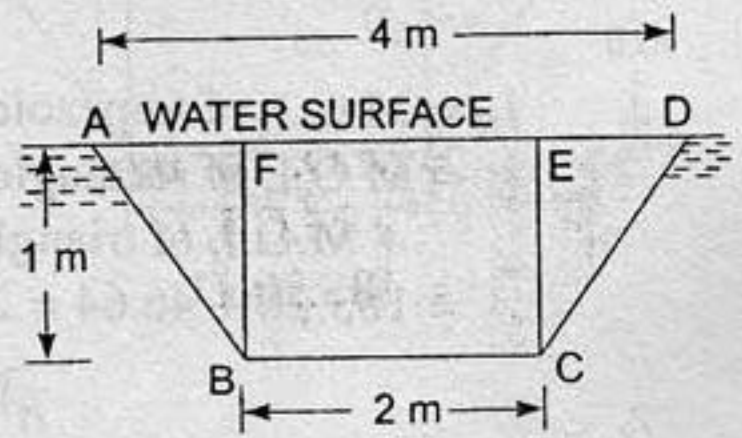
$$h^* = \frac{229.846}{78 \times 2.769} + 2.769 \qquad (\because \quad A = 78 \text{ m}^2)$$

$$= \mathbf{3.833 \text{ m}}$$

Problem 3.9 *A trapezoidal channel 2 m wide at the bottom and 1 m deep has side slopes 1 : 1. Determine :*

(i) the total pressure, and

(ii) the centre of pressure on the vertical gate closing the channel when it is full of water.



Width at bottom $= 2$ m

Depth, $d = 1$ m

Side slopes $= 1 : 1$

$\therefore$ Top width, $AD = 2 + 1 + 1 = 4$ m

Area of rectangle *FBEC*, $A_1 = 2 \times 1 = 2 \text{ m}^2$

Area of two triangles *ABF* and *ECD*, $A_2 = \dfrac{(4-2)}{2} \times 1 = 1 \text{ m}^2$

$\therefore$ Area of trapezoidal *ABCD*, $A = A_1 + A_2 = 2 + 1 = 3 \text{ m}^2$

Depth of C.G. of rectangle *FBEC* from water surface,

$$\bar{h}_1 = \frac{1}{2} = 0.5 \text{ m}$$

Depth of C.G. of two triangles *ABF* and *ECD* from water surface,

$$\bar{h}_2 = \frac{1}{3} \times 1 = \frac{1}{3} \text{ m}$$

$\therefore$ Depth of C.G. of trapezoidal *ABCD* from free surface of water

$$\bar{h} = \frac{A_1 \times \bar{h}_1 + A_2 \times \bar{h}_2}{(A_1 + A_2)} = \frac{2 \times 0.5 + 1 \times 0.33333}{(2 + 1)} = .44444$$

(i) **Total Pressure (F).** Total pressure F is given by

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 3.0 \times 0.44444 = \mathbf{13079.9\ N.}$$

(*ii*) **Centre of Pressure (h*).** M.O.I. of rectangle *FBCE* about its C.G.,

$$I_{G_1} = \frac{bd^3}{12} = \frac{2 \times 1^3}{12} = \frac{1}{6} \text{ m}^4$$

M.O.I. of *FBCE* about an axis passing through the C.G. of trapezoidal

or $$I_{G_1}^* = I_{G_1} + A_1 \times [\text{Distance between C.G. of rectangle and C.G. of trapezoidal}]^2$$

$$= \frac{1}{6} + 2 \times \left[\bar{h}_1 - \bar{h}\right]^2$$

$$= \frac{1}{6} + 2 \times [0.5 - .4444]^2 = .1666 + .006182 = 0.1727$$

M.O.I. of the two triangles *ABF* and *ECD* about their C.G.,

$$I_{G_2} = \frac{bd^3}{36} = \frac{(1+1)\times 1^3}{36} = \frac{2}{36} = \frac{1}{18} \text{ m}^4.$$

M.O.I. of the two triangles about the C.G. of trapezoidal,

$$I_{G_2}{}^* = I_{G_1} + A_2 \times [\text{Distance between C.G. of triangles and C.G. of trapezoidal}]^2$$

$$= \frac{1}{18} + 1\times\left[\bar{h} - \bar{h}_2\right]^2 = \frac{1}{18} + 1\times\left[.4444 - \frac{1}{3}\right]^2$$

$$= \frac{1}{18} + (.1111)^2 = 0.0555 + (.1111)^2$$

$$= .0555 + 0.01234 = 0.06789 \text{ m}^4$$

∴ M.O.I. of the trapezoidal about its C.G.

$$I_G = I_{G_1}* + I_{G_2}* = .1727 + .06789 = 0.24059 \text{ m}^4$$

Then centre of pressure (h^*) on the vertical trapezoidal,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{0.24059}{3 \times .4444} + .4444 = 0.18046 + .4444 = 0.6248$$

$$\simeq \mathbf{0.625\ m.}$$

Alternate Method

The distance of the C.G. of the trapezoidal channel from surface AD is given by

$$x = \frac{(2a+b)}{(a+b)} \times \frac{h}{3} = \frac{(2 \times 2 + 4)}{(2+4)} \times \frac{1}{3} \qquad (\because \quad a = 2, b = 4 \text{ and } h = 1)$$

$$= 0.444 \text{ m}$$

$$\bar{h} = x = 0.444 \text{ m}$$

Total pressure, $F = \rho g A \bar{h} = 1000 \times 9.81 \times 3.0 \times .444$

$= \mathbf{13079\ N}$

Centre of pressure, $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$$I_G = \frac{(a^2 + 4ab + b^2)}{36(a+b)} \times h^3 = \frac{(2^2 + 4 \times 2 \times 4 + 4^2)}{36(2+4)} \times 1^3 = \frac{52}{36 \times 6} = 0.2407 \text{ m}^4$$

$$h^* = \frac{0.2407}{3.0 \times .444} + .444 = \mathbf{0.625 \text{ m}}.$$

Problem 3.11 *A tank contains water upto a height of 0.5 m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water upto 1 m height. Calculate :*

(i) total pressure on one side of the tank,

(ii) the position of centre of pressure for one side of the tank, which is 2 m wide.

Depth of water	$= 0.5$ m
Depth of liquid	$= 1$ m
Sp. gr. of liquid	$= 0.8$
Density of liquid,	$\rho_1 = 0.8 \times 1000 = 800 \text{ kg/m}^3$
Density of water,	$\rho_2 = 1000 \text{ kg/m}^3$
Width of tank	$= 2$ m

(*i*) **Total pressure on one side** is calculated by drawing pressure diagram, which is shown in Fig. 3.14.

Intensity of pressure on top, $p_A = 0$

Intensity of pressure on D (or DE), $p_D = \rho_1 g . h_1$

$$= 800 \times 9.81 \times 1.0 = 7848 \text{ N/m}^2$$

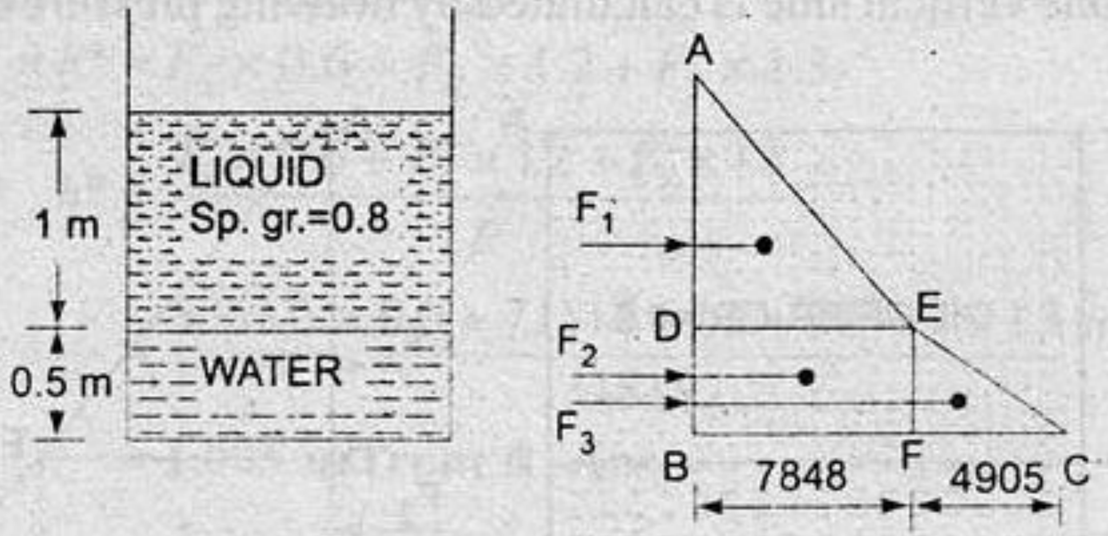


Fig. 3.14

Intensity of pressure on base (or BC), $p_B = \rho_1 g h_1 + \rho_2 g \times 0.5$

$$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905 = 12753 \text{ N/m}^2$$

Now force $F_1 = \text{Area of } \Delta ADE \times \text{Width of tank}$

$$= \frac{1}{2} \times AD \times DE \times 2.0 = \frac{1}{2} \times 1 \times 7848 \times 2.0 = 7848 \text{ N}$$

Force $F_2 = \text{Area of rectangle } DBFE \times \text{Width of tank}$

$$= 0.5 \times 7848 \times 2 = 7848 \text{ N}$$

$F_3 = \text{Area of } \Delta EFC \times \text{Width of tank}$

$$= \frac{1}{2} \times EF \times FC \times 2.0 = \frac{1}{2} \times 0.5 \times 4905 \times 2.0 = 2452.5 \text{ N}$$

Total pressure, $F = F_1 + F_2 + F_3$

$$= 7848 + 7848 + 2452.5 = \mathbf{18148.5 \text{ N}}.$$

(*ii*) **Centre of Pressure (h*).** Taking the moments of all force about A, we get

$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2 (AD + \frac{1}{2} BD) + F_3 [AD + \frac{2}{3} BD]$$

$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848 \left(1.0 + \frac{0.5}{2}\right) + 2452.5 \left(1.0 + \frac{2}{3} \times .5\right)$$

$$= 5232 + 9810 + 3270 = 18312$$

$$h^* = \frac{18312}{18148.5} = \mathbf{1.009 \text{ m from top}}$$